

## 2. BASIC ELECTROSTATICS

### 2.1 INTRODUCTION

Electrostatics is associated with materials on which electrical charge moves only slowly (insulating materials) and with electrically isolated conductors. 'Insulation' and 'isolation' prevents easy migration of charge. So charges stay in place (are 'static'). It is the effects that the charges produce that are then important.

The parameter of basic importance in electrostatics is 'charge'. It is **charge** which gives rise to the electric fields generating forces that attract thin films and particles to surfaces and **charge** which gives rise to high voltages in low capacitance systems. High voltages can cause sparks. If these have sufficient energy they may ignite flammable gases, cause shocks to personnel, damage semiconductor devices and upset the operation of microelectronic equipment.

This section introduces basic concepts of electrostatics and provides a framework for understanding and quantitatively assessing electrostatic questions and problems. A good textbook is Feynman 'Lectures on Physics' [1]. Other good sources are available [2,3].

### 2.2 CHARGE

Static electricity arises as the separation of positive and negative charges at the interface between two dissimilar surfaces. If one or other of the surfaces prevent easy migration of charge, or the conductor on which they reside is isolated, then this charge is 'static' on the surface and remains available to influence the surroundings. 'Static' electricity can also arise on surfaces as trapped ions from the air.

Static charges may be electrons, or positive, or negative ions - but they are in the basic units of electronic charge  $1.602 \times 10^{-19}$  coulomb. On a surface there are some  $10^{19}$  atomic sites per  $\text{m}^2$  - so if the charge of even quite a small fraction of the surface atoms is changed then quite large quantities of charge are easily involved.

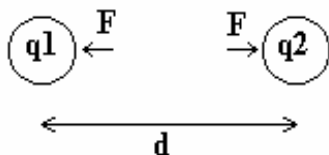
### 2.3 FORCE BETWEEN CHARGES

The force between two charges,  $Q_1$  and  $Q_2$ , is proportional to the quantities of charge, and inversely proportional to the square of the separation distance,  $d$ :

$$F = - Q_1 Q_2 / (4 \pi \epsilon_0 d^2)$$

This is Coulomb's Law. The force  $F$  is in Newtons, the quantities of charge  $Q_1$  and  $Q_2$  are in Coulombs and the separation distance  $d$  is in meters. The constant  $\epsilon_0$  is called the 'permittivity of free space' and has a value  $8.854 \times 10^{-12}$  (see later). Charges of the same polarity repel each other and of opposite polarity attract.

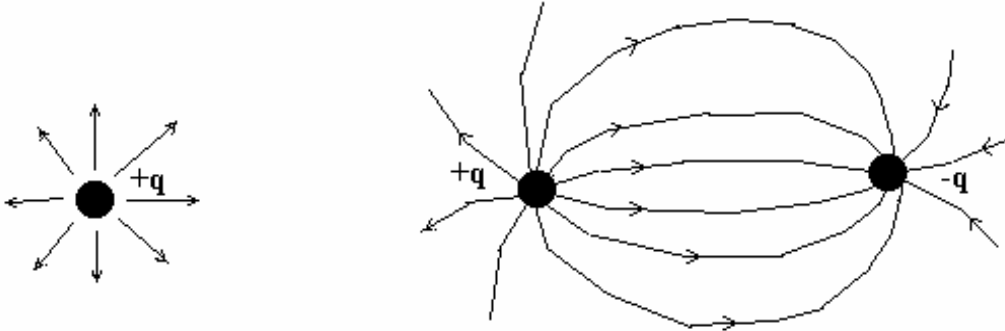
Examples of electrostatic forces are the attraction between thin layers of plastic film and of fine dust particles to surfaces.



## 2.4 ELECTRIC FIELD

Around a charged body there is a force of attraction or repulsion for any other charges. This force, for a unit of charge, at any point is called the 'electric field',  $E$ , at that point.

The direction of the field at any point depends on the direction of force on a positive charge there - and is hence a vector parameter. Diagrammatically it is useful to draw 'lines of force' to represent the electric field. The closer together the lines of force the stronger the electric field.



From Coulombs Law the electric field strength  $E$  at a distance  $d$  from a charge  $Q$  will be:

$$E = Q / (4 \pi \epsilon_0 d^2)$$

The constant of proportionality  $1 / (4 \pi \epsilon_0)$  is, for historical reasons, defined in the MKS system of units by the relation:

$$1 / (4 \pi \epsilon_0) = c^2 \times 10^{-7}$$

The constant  $\epsilon_0$  hence has a value  $8.854 \times 10^{-12}$  coulomb<sup>2</sup>/(Newton m<sup>2</sup>) or coulomb / (volt m). This is the 'permittivity of free space' noted above.

## 2.5 POTENTIAL

The 'potential' at a point is defined as the amount of work needed to bring a unit charge from infinity to that point. The potential difference between two points is then the work done to move a unit charge between these two points. The work done does not depend on the route followed so the potential is a scalar quantity.

The unit of potential is the 'volt'. The potential at a radial distance  $d$  from a single point charge  $Q$  is:

$$V = Q / (4 \pi \epsilon_0 d)$$

The electric field relates to potential as:

$$E = -\nabla V$$

## 2.6 ELECTRIC FLUX DENSITY

The electric field from an isolated point charge is radial. The field may conveniently be represented diagrammatically by lines of force. The 'flow' or flux of electric field across any spherical surface enclosing a point charge  $Q$  will then be  $Q/\epsilon_0$ .

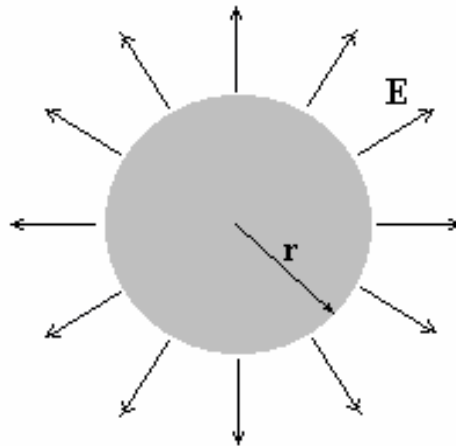
## 2.7 GAUSS' LAW

Gauss' Law states that the nett flux through any surface equals the nett charge enclosed within that surface.

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

For a uniform distribution density of charge  $\rho$  ( $\text{C m}^{-3}$ ) in a sphere of radius  $r$  the total charge is  $\frac{4}{3} \pi r^3 \rho$ . The outward flux will be  $4 \pi r^2 E$ . Hence the electric field at the sphere surface is:

$$E = \rho r / (3 \epsilon_0)$$

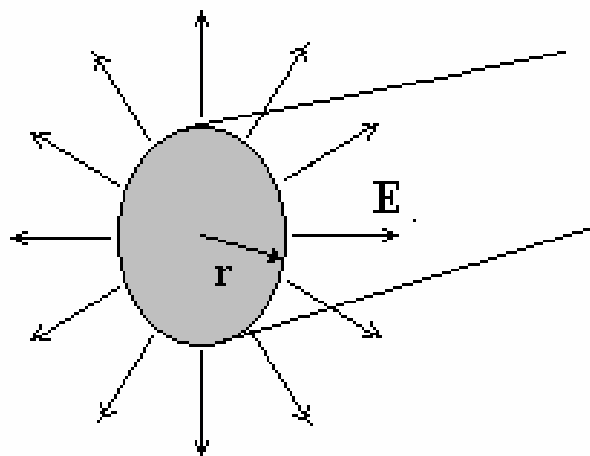


The maximum voltage at the centre of such a spherical distribution of charge is:

$$V = \rho r^2 / (6 \epsilon_0)$$

Similarly for a cylindrical distribution of charge density:

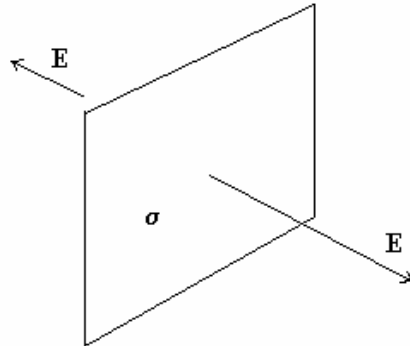
$$E = \rho r / (2 \epsilon_0) \text{ and } V = \rho r^2 / (4 \epsilon_0)$$



## 2.8 INDUCED CHARGE

The electric field from each side of a sheet of uniform charge density  $\sigma$  ( $\text{C m}^{-2}$ ) is:

$$E = \sigma / (2 \epsilon_0)$$



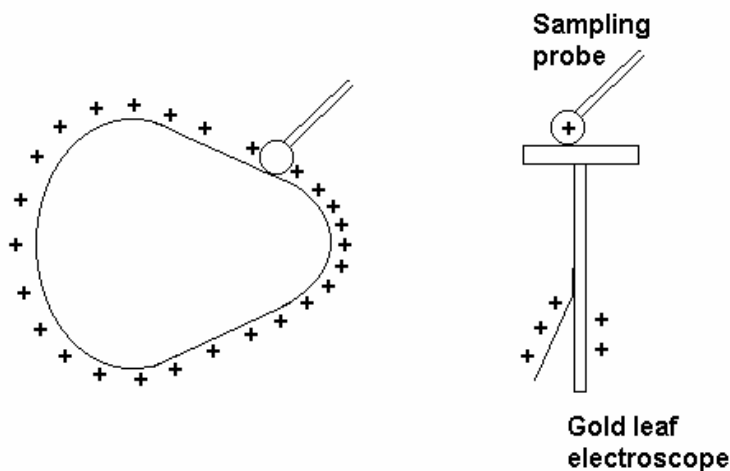
There can be no electric fields inside a conductor and the electric field outside is:

$$E = \sigma / \epsilon_0$$

Thus a conducting body placed in an electric field will have charge 'induced' on it with the density of charge proportional to the electric field.

The charge induced on a conductor is of opposite polarity to that of the charge source. Opposite charges attract so there is an attraction between a charge and a conducting surface. For a plane conducting surface this 'image charge' is an equal distance behind the surface so calculation of the force involves a distance equal to twice the separation distance from the conducting surface.

If a small conducting body (e.g. a small sphere) on an insulating handle is touched to a conducting body charge will be shared and the probe will acquire a charge dependent of the local electric field - related to the local surface charge density. If the amount of charge can be tested then we have a means to check if the conducting body was charged and to explore the density of charge over that body. A 'gold leaf electroscope' indicates charge by repulsion between a fine gold leaf from it's nearby conducting support and gives an angle of the leaf indicating the level of charge. Sharing charge from a conducting probe with a gold leaf electroscope provides a good classical method to indicate and measure charge. Such observations show that the electric field (and charge density) on a conducting body is inversely proportional to the local radius of curvature.



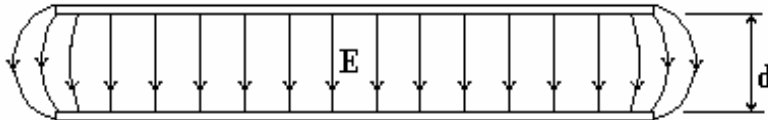
## 2.9 CAPACITANCE

For two conducting plates of area  $A$  separated by a distance  $d$  (m) with a voltage difference between them  $V$  the electric field between the plates is:

$$E = V / d$$

and the quantity of charge on each of the plates:

$$Q = A \sigma = \epsilon_0 A E = \epsilon_0 A V / d$$



The proportionality between the quantity of charge  $Q$  and the voltage  $V$  is called the 'capacitance'  $C$  of the system. The unit of capacitance is the 'farad' with units of coulomb/volt. For the pair of parallel plates:

$$C = Q/V = \epsilon_0 A / d$$

This relation is not quite accurate because the electric field is not uniform near the edges of the plates and extends outwards from them a bit.

For an isolated sphere of radius  $a$  (m) the capacitance is  $4 \pi \epsilon_0 a$  - for a radius of 10mm this is about 1 picofarad. The capacitance for a number of simple geometric arrangements (for instance concentric spheres, concentric cylinders, parallel rods and rods near plane conductors) can be calculated analytically [2,35]. These can provide useful guidance for estimating capacitance in practical situations. Alternatively capacitance values may be calculated by computer modelling calculations [36,37] or, with suitable care, measured.

## 2.10 ENERGY

The electrostatic energy  $U$  (joule) of a capacitor of  $C$  (farads) is:

$$U = 0.5 C V^2$$

## 2.11 POISSON'S EQUATION

$$\nabla^2 V = -\rho/\epsilon_0$$

- where  $\rho$  is the volume density of charge ( $C m^{-3}$ ). If no charges are present then Poisson's equation becomes Laplace's equation:

$$\nabla^2 V = 0$$

There are a number of geometric forms for which the above equations can be solved analytically. Two and three dimensional finite element and finite difference computer modelling programs are available to find potential and electric field distributions within practical geometric arrangements [36,37].

## 2.12 DIELECTRICS

If an insulator is placed between the plates of a parallel plate capacitor the capacitance is increased.



If a block of conductor were placed between the capacitor plates the field in the remaining gap would be increased. A similar effect occurs with insulating materials, but because charges cannot move through or over the surface of insulators the material becomes 'polarised'. The atoms or molecules distort slightly under the electrical stress, polarise, and while these effects cancel out within the insulator they appear as surface charges on either side of the block of material.

Applying Gauss's Law to the surface S the electric field in the insulator is:

$$E = (\sigma_f - \sigma_p) / \epsilon_0$$

where  $\sigma_f$  and  $\sigma_p$  are the charge densities on the free surface of the capacitor plates and the surface of the insulator. The polarisation charges are proportional to the electric field and this is usually written:

$$P = X \epsilon_0 E = \sigma_p$$

$$E = \sigma_f / (\epsilon_0 (1 + X))$$

where X is the 'electric susceptibility' of the insulator/dielectric. This shows the reduction of the electric field within the insulator compared to that just outside.

The total charge on the capacitor plates is  $\sigma_f A$  so the capacitance by  $Q = CV$  becomes for a capacitor fully filled by an insulator:

$$C = \sigma_f A / V = A E (\epsilon_0 (1 + X)) / (E d)$$

$$C = \epsilon_0 A (1 + X) / d$$

The increase in capacitance for this fully filled capacitor is the factor (1+X) and this is usually called the 'permittivity' of the material - k.

## 2.13 CHARGE MIGRATION AND DISSIPATION

Insulating materials have very few free electrical charges so only very small currents flow under the influence of an electric field. In more usual terms the volume 'resistivity' may be from  $10^{12}$  to  $10^{16}$  ohm m and the surface resistivity over  $10^{10}$  ohms per square. Not only are there few charge carriers but their number and the velocity of their movement may depend in a non-linear way upon the electric field. In this situation 'resistivity' is not a very useful parameter for characterising materials. Of more direct practical importance in the control of

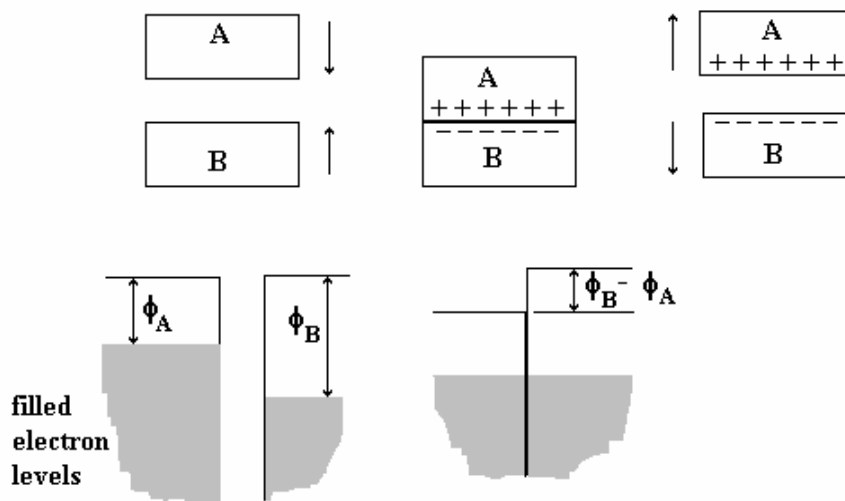
static is the timescale for static charge dissipation. If the timescale for charge dissipation is less than the timescale for charge generation then no significant charge levels or surface voltages will arise and there will be no static problems.

For uniform materials, such as liquids, it is expected that the time for charge dissipation  $\tau$  ( $\sigma$ ) can be related to the volume resistivity  $\rho$  (ohms) and permittivity  $k$  as:

$$\tau = k \epsilon_0 \rho$$

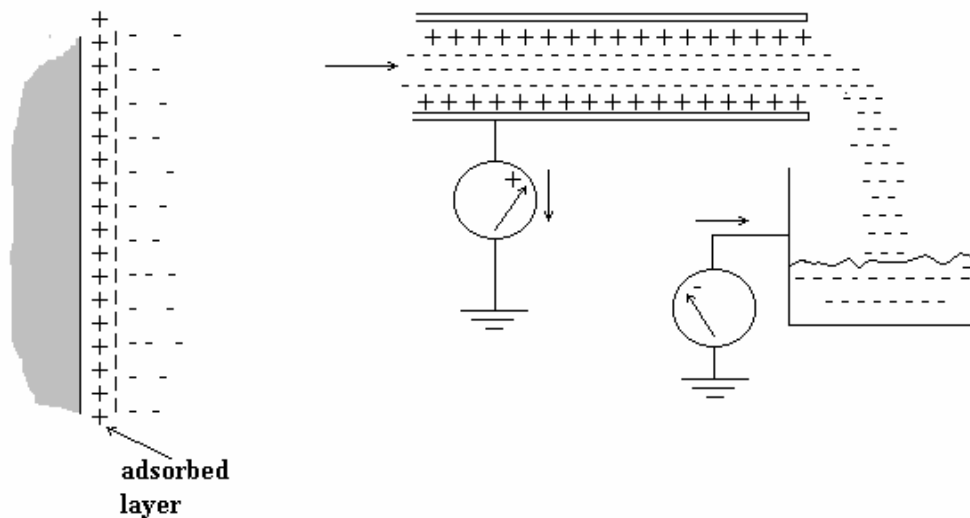
## 2.14 CHARGE SEPARATION MECHANISMS

Contact between surfaces results in transfer of charge as electrons to match up energy levels of neighbouring atoms around the points of contact. The flow of charge continues until inhibited by the potential difference of the 'contact potential' of the surfaces. On separation the surfaces may retain the transferred charge.



Charge transfer depends on differences in 'contact potential' and materials can be listed as a 'triboelectric series'. Charge separation depends on surface conditions. It is strongly affected by contamination and by the speed and pressure of rubbing actions. Only in a few instances, for example in photocopiers and in some paint spray situation, are triboelectric effects used to control charging in a defined way.

Charge separation also occurs when a liquid meets a conducting surface. Ions formed by dissociation in the fluid or from trace impurities will adsorb at the boundary surface. This creates the so-called 'double layer' at the boundary. An appreciable charge can be adsorbed to the surface - which is matched by charge in the fluid. With relatively insulating fluids the charge in the volume can be swept away by fluid flow as a 'streaming current' which can be in the range  $10^{-10}$  to  $10^{-7}$  A. A double layer also occurs when water breaks away from a surface - and this is responsible for the formation of the highly charged mists that can arise when high pressure water jets hit surfaces.



## 2.15 ELECTRICAL BREAKDOWN

If an electric field, in air at normal temperature and pressure, increases above about  $3 \text{ MV m}^{-1}$  then 'electrical breakdown' occurs. A loose electron or ion accelerates in the local electric field and if it picks up enough energy from the field before it hits another air molecule it will cause ionisation of that molecule. This can lead to further ionisation and rapid growth of the number of ions present. This multiplication of ionisation can destroy the insulating properties of the air and lead to passage of a high current accompanied by the emission of heat and light and sound - this is a spark discharge. A good textbook and reference book on electrical breakdown of gases is that by Meek and Craggs [4].

If the electric field exceeds the critical value for ion multiplication in just a localised region, for example near a sharp point, while the average electric field over the gap to nearby conductors is quite low, then a localised discharge takes place with a fairly low current (up to perhaps a mA) between the electrodes. This is a 'corona' discharge. If the electric field exceeds the breakdown strength of air over several mm, for example with an electrode of several mm radius of curvature, then the discharge involves more current and has a condensed luminous channel which does not reach to another electrode. This is a 'brush discharge'.

If the breakdown strength of air is exceeded near an electrode and also the average electric field over the gap to the other electrode exceeds about  $500 \text{ kV m}^{-1}$  then breakdown is likely to occur as a spark discharge between the electrodes. The voltage for electrical breakdown depends on the geometry of the gap, on the gas and on the pressure. As the pressure or gap is reduced the breakdown voltage decreases to a minimum and then rises - following the Paschen Law curve [2,4]. For air the minimum is about 300 volts.

If a high electric field arises at the surface of a fluid the surface will distort and break up [5]. This occurs at a lower electric field than that for air breakdown.

## REFERENCES

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